

Functions

Kangaroo 2017

19. The sequence of functions $F_1(x), F_2(x), \dots$ satisfies the following conditions:

$$F_1(x) = x, \quad F_{n+1}(x) = \frac{1}{1 - F_n(x)}.$$

The integer C is a three-digit cube such that $F_C(C) = C$.

What is the largest possible value of C ?

Kangaroo 2016

16. A function $f(x)$ has the property that, for all positive x , $3f(x) + 7f\left(\frac{2016}{x}\right) = 2x$.

What is the value of $f(8)$?

BMO 1 2014

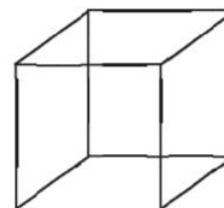
6. Determine all functions $f(n)$ from the positive integers to the positive integers which satisfy the following condition: whenever a, b and c are positive integers such that $1/a + 1/b = 1/c$, then

$$1/f(a) + 1/f(b) = 1/f(c).$$

Logic

Kangaroo 2017

10. At each of the vertices of a cube sits a Bunchkin. Two Bunchkins are said to be adjacent if and only if they sit at either end of one of the cube's edges. Each Bunchkin is either a 'truther', who always tells the truth, or a 'liar', who always lies. All eight Bunchkins say 'I am adjacent to exactly two liars'. What is the maximum number of Bunchkins who are telling the truth?



Kangaroo 2015

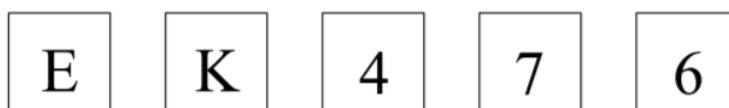
13. Twenty-five people who always tell the truth or always lie are standing in a queue. The man at the front of the queue says that everyone behind him always lies. Everyone else says that the person immediately in front of them always lies. How many people in the queue always lie?

Kangaroo 2014

15. A box contains seven cards numbered from 301 to 307. Graham picks three cards from the box and then Zoe picks two cards from the remainder. Graham looks at his cards and then says "I know that the sum of the numbers on your cards is even". What is the sum of the numbers on Graham's cards?

Kangaroo 2012

4. Five cards are laid on a table, as shown. Every card has a letter on one side and a number on the other side.
Peter says: "For every card on the table, if there is a vowel on one side of the card, then there is an even number on the other side."
What is the smallest number of cards Sylvia must turn over in order to be certain that Peter is telling the truth?



Integers 1

Kangaroo 2015

20. The number $N = 3^{16} - 1$ has a divisor of 193. It also has some divisors between 75 and 85 inclusive. What is the sum of these divisors?

Kangaroo 2012

8. The three prime numbers a , b and c are such that $a > b > c$, $a + b + c = 52$ and $a - b - c = 22$.
What is the value of abc ?

Kangaroo 2013

14. What is the largest three-digit integer that can be written in the form $n + \sqrt{n}$ where n is an integer?

Kangaroo 2013

10. In the equation $2^{m+1} + 2^m = 3^{n+2} - 3^n$, m and n are integers. What is the value of m ?

Integers 2 - Advanced

GBMO 2015

5. (a) The integer N is a square. Find, with proof, all possible remainders when N is divided by 16.
- (b) Find all positive integers m and n such that

$$m! + 76 = n^2.$$

[The notation $m!$ stands for the factorial of m , that is, $m! = m \times (m - 1) \times \cdots \times 2 \times 1$. For example, $4! = 4 \times 3 \times 2 \times 1$.]

GBMO 2017

2. A positive integer is said to be *jiggly* if it has four digits, all non-zero, and no matter how you arrange those digits you always obtain a multiple of 12.

How many jiggly positive integers are there?

BMO 1 2011

1. Find all (positive or negative) integers n for which $n^2 + 20n + 11$ is a perfect square. *Remember that you must justify that you have found them all.*

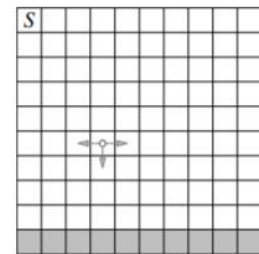
Combinatorics

Kangaroo 2017

14. The set S is given by $S = \{1, 2, 3, 4, 5, 6\}$. A non-empty subset T of S has the property that it contains no pair of integers that share a common factor other than 1. How many distinct possibilities are there for T ?

GBMO 2015

3. A ladybird is going for a wander around a 10×10 board, subject to the following three rules (see the diagram).
- (i) She starts in the top left cell, labelled S .
 - (ii) She only moves left, right or down, as indicated.
 - (iii) She never goes back to a cell that she has already visited.



In how many different ways can she reach the bottom row of cells, shaded grey?

Kangaroo 2012

20. How many subsets of the set $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ exist in which the sum of the largest element and the smallest element is 11?

BMO 1 2013

4. Isaac is planning a nine-day holiday. Every day he will go surfing, or water skiing, or he will rest. On any given day he does just one of these three things. He never does different water-sports on consecutive days. How many schedules are possible for the holiday?